

## Combined Matrix Transformations

Eg Here are two transformations.

**A** is rotation by  $90^\circ$  anti-clockwise about the origin.

**B** is a reflection in the line  $y = x$ .

Use matrix multiplication to work out the single matrix that performs these two operations – **A** and then **B**.

*How do I do this?*

Write down the single matrix that represents transformation **A**.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Write down the single matrix that represents transformation **B**.

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Write down the multiplication to be performed. Remember that the first transformation is the last one in the multiplication, so the order is reversed.

$$\mathbf{BA} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Multiply the two matrices together

$$\mathbf{BA} = \begin{pmatrix} (0 \times 0 + 1 \times (-1)) & (0 \times 1 + 1 \times 0) \\ (1 \times 0 + 0 \times (-1)) & (1 \times 1 + 0 \times 0) \end{pmatrix}$$

Find the matrix

$$\mathbf{BA} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$



### List of basic matrix transformations

Rotation by 90° anti-clockwise or 270° clockwise around the origin

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Rotation by 180° anti-clockwise around the origin

$$\mathbf{B} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Rotation by 270° anti-clockwise or 90° clockwise around the origin

$$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Reflection in the line  $x=0$

$$\mathbf{D} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Reflection in the line  $y=0$

$$\mathbf{E} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Reflection in the line  $y=x$

$$\mathbf{F} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Reflection in the line  $y=-x$

$$\mathbf{G} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$



## Practice Questions

- 1 Rotation anticlockwise  $180^\circ$  followed by a reflection in the line  $x=0$
- 2 Reflection in the line  $x=0$  followed by a rotation of  $180^\circ$
- 3 Rotation  $90^\circ$  clockwise followed by a reflection in the line  $y=x$
- 4 Rotation  $90^\circ$  anti-clockwise followed by a reflection in the line  $y=x$
- 5 Reflection in the line  $y=0$  followed by a rotation of  $180^\circ$
- 6 Rotation  $90^\circ$  clockwise followed by a reflection in the line  $y=-x$
- 7 Rotation  $90^\circ$  anti-clockwise followed by a reflection in the line  $y=-x$
- 8 Rotation of  $90^\circ$  clockwise followed by a reflection in the line  $y=0$  followed by a rotation of  $90^\circ$  anti-clockwise.
- 9 Reflection in the line  $y=0$  followed by a reflection in the line  $y=x$  followed by a reflection in the line  $x=0$
- 10 Rotation about the origin of  $90^\circ$  clockwise followed by a reflection in the line  $y=-x$  followed by a rotation of  $180^\circ$
  
- 11 What is the image of the matrix  $\begin{pmatrix} 3 & 4 \\ 5 & -2 \end{pmatrix}$  when it has been reflected in the line  $x=0$  and then rotated by  $90^\circ$  anti-clockwise?
- 12 What is the image of the matrix  $\begin{pmatrix} 2 & -1 \\ 4 & 7 \end{pmatrix}$  when it has been rotated  $90^\circ$  clockwise and then reflected in the line  $y=x$ .

